Exact Solution Scientific Consulting LLC

From Concept to Product

Aerospace Vehicle "Inclusive" Dynamics Six Degree of Freedom Modeling

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AViD-6DOF, A comprehensive simulation of space vehicle and other space object dynamics including collisions and break-up (separation events).

For more information contact: Dr. David R. Bergman, davidrbergman@essc-llc.com

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1. Introduction

ESSC-LLC has been engaged for some time in the development of specialized tracking filters and event identifiers for use in ballistic missile and satellite tracking systems. Two specific areas of interest are (1) satellites interacting with debris field and (2) off nominal break up of missiles [1-5]. Both of these pursuits are expected to lead to the development of enhanced situational awareness (SA) for both military and civilian organizations. ESSC-LLC is involved in both the development of algorithms for detecting potential threat events and event reconstruction, and the development of integrated SA user interfaces complete with visual display, alerts, status *etc*.

As research and development of such products progressed the need became apparent for a high fidelity 6 degree of freedom (6DOF) motion simulator for a variety of aerospace vehicles moving under the influence of gravity, aero forces and their own thrust capabilities. This simulator also required replication of separation events including off nominal break up events due to a variety of failure scenarios and hard body collisions. Hence the development of the Aerospace Vehicle "Inclusive" Dynamics 6 Degree of Freedom, (AV*i*D-6DOF), model. The name is mostly self-explanatory and is somewhat borrowed from Zipfel's text [6]. The term "Inclusive" is added as an indicator of our intent for this to grow into an all-inclusive motion simulator designed to handle events that are not normally included in a 6DOF simulator, *e.g.* collisions, stress-strain analysis of the moving bodies, especially during collisions and other jarring events that could cause changes in the moment of inertia of the body and its aerodynamic properties or destruction of a parent body into multiple pieces.

Though AV*i*D-6DOF is intended to manage all data required for a complete analysis of a moving extended body we cannot predict all varieties of environment that may be required. Hence this white paper will cover the dynamics modeling without specific reference to environmental models, *e.g.* aerodynamics. The details of this aspect of the model require information regarding the nature of the fluid(s) defining the environment and the airfoil of the vehicle, or a profile of the vehicle body. In many cases this interaction between the moving body and the fluid can be expressed in terms of drag and lift coefficients and is left to the user to define via look up tables or as a plug-in function.

AV*i*D-6DOF is an internal research tool used by ESSC-LLC in support of the previously mentioned research and development efforts. As with previously published ESSC-LLC white papers this is not meant to be a comprehensive user manual or design document. The reader should get a good idea of the type of research ESSC-LLC is engaged in and the types of proprietary simulation tools that have been developed by ESSC-LLC in support of that research.

2. Object modeling

Objects used by AV*i*D-6DOF contain enough information to feed the 6DOF ordinary differential equation (ODE) solver to determine the translational and rotational states of the object relative to a global frame of reference. Variables required by objects and their units are listed in table 1.

Variable	Symbol	units	Notes
Body Name	name	N/A	Users can specify a unique name as an identifier for each object.
Mass	Mass	kg	-
Basic Shape Parameters	B	struct	This is a sub-structure which allows one to specify parameters used in defining a basic shape; cone, cylinder, sphere, *etc*.
Moment of inertia (*x*-principal axis)	Ixx	$kg*m^2$	-
Moment of inertia (*y*-principal axis)	Iyy	$kg*m^2$	-
Moment of inertia (*z*-principal axis)	Izz	$kg*m^2$	-
x-position Body	xb	m	These parameters can be arrays of values describing points on the surface of the body. This is particularly useful for bodies with a high degree of symmetry. All points are referenced in the body frame with origin at the center of mass.
y-position Body	yb	m	
z-position Body	zb	m	

Table 1 *Object variables*

AV*i*D-6DOF does not directly handle CAD models or other facet models which describe the physical body. These may be used in force and moment calculators called by AV*i*D-6DOF during propagation but a physical representation of the body is not necessary otherwise. For plotting purposes and visualization AV*i*D-6DOF will allow a basic shape or shape parameters to be attributed to each specific object. The kinematics state of a body is not included in this data. AV*i*D-6DOF manages the association between objects and state data internally.

3. Trajectory modeling

Our use of the term trajectory is broader than a time parameterized curve in 3-d space. AV*i*D-6DOF predicts the complete state of an extended tri-inertial body in space acted upon by external forces and torques (or moments). Hence at each point in time we are producing a minimum of 12 variables, 3 Cartesian coordinates and their generalized velocities (or momentum) and 3 Euler angles (or pitch, roll and yaw) and their corresponding generalized velocities [7]. For the purposes

For more information contact: Dr. David R. Bergman, davidrbergman@essc-llc.com

of testing tracking filters against ground truth AV*i*D-6DOF also provides acceleration as part of the state for a total of 18 variables. All of these state variables are referenced to a global frame of reference.

3.1 Translational dynamics

Translational motion is governed by Newton's second law. The specification of a frame of reference is crucial to the implementation of Newton's second law. While the standard equation, $\vec{F}_{ext}^{Net} = m\vec{a}$, is valid in any inertial frame of reference (non-accelerating and non-rotating) it is sometimes easier to model vehicle motion in a reference frame that is fixed to rotate with the earth, referred to as Earth Centered Earth Fixed (ECEF) or ECF for short. The use of Earth Centered Inertial (ECI) seems like a better choice for implementing Newton's laws but requires tracking the earth's rotation during the simulation along with everything else. Developing Newton's second law in an accelerating frame of reference requires the introduction of "Fictitious Forces" to account for the acceleration of the reference frame. This is a well-known device and can be found in any text on mechanics [7-10]. AV*i*D-6DOF is capable of running simulations in either ECI or ECF for a rotating earth, and allows the user to specify a non-rotating or rotating earth model. Figure 1 shows the definitions of these frames of reference and their relation to each other relative to an exaggerated ellipsoid with $R_{pol}/R_{eq} = 0.8$ and scaled to $R_{eq} = 1.0$.

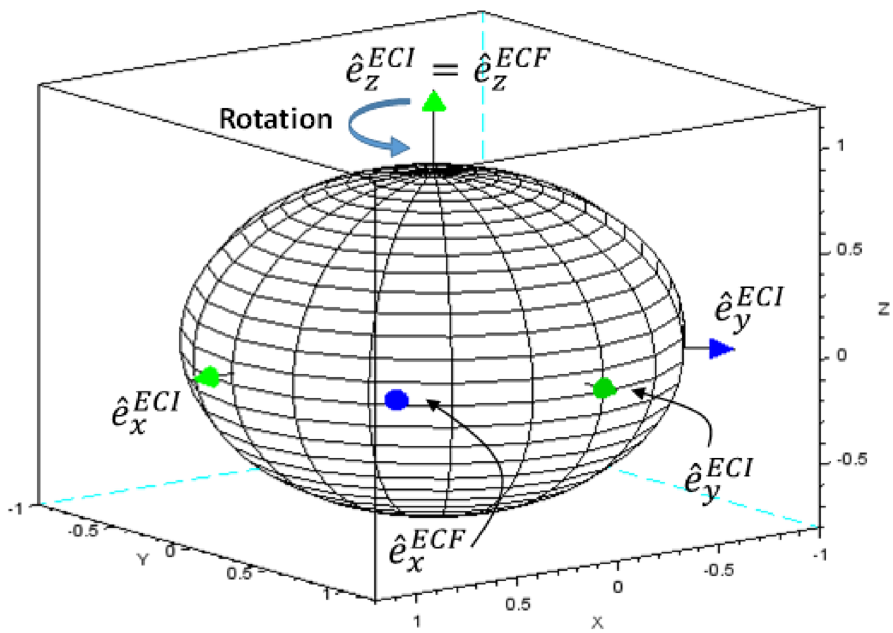

Figure 1 *ECI and ECF coordinates used for expressing Newton's law of motion*

3.2 Attitude dynamics

AV*i*D-6DOF contains two sets of attitude dynamics algorithms, one based on Euler angles and the other based on quaternions [6, 7]. Moments are determined by the interaction of the body and the environment. The rotational equations for the angular rate, $\vec{\omega}$, are expressed in the internal body frame and from the solution at each step the orientation of the body axes relative to global coordinates is determined. Figure 2 shows a generic inertial ellipsoid with three non-equal principal moments of inertia along with rotational dynamic variables referenced to a global frame.

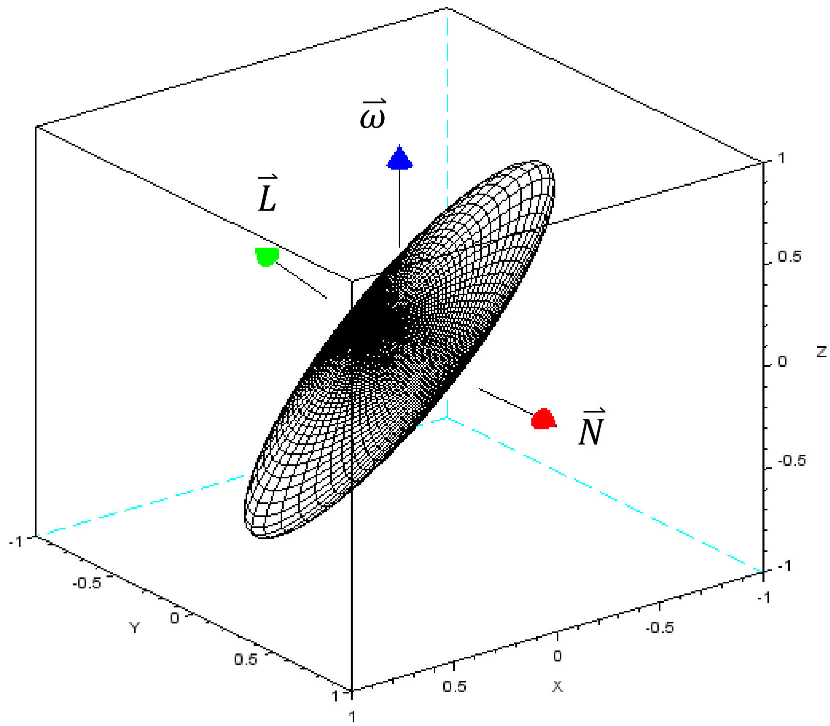

Figure 2 *Example of a tri-inertial ellipsoid and state variables in its body frame*

Represented in the figure are the body's angular velocity, $\vec{\omega}$, angular momentum, $\vec{L}$ and a torque or moment vector, $\vec{N}$. The body axes are not shown but would lie along the principal axes of the ellipsoid. The inertial ellipsoid is scaled so the largest axis is 1 and (*x*, *y*, *z*) is a global frame of reference with origin at the center of the ellipsoid.

In addition to being capable of propagating the rotational equations of motion AV*i*D-6DOF contains implementations of known exact solutions for freely rotating bodies suitable in cases of torque free motion [10].

4. External force and moment modeling

All forces are modeled in separate functions. AV*i*D-6DOF calls each function for updates to the forces acting on the body as its state is propagated. As such there is really no limit to what can be modeled in AV*i*D-6DOF as long as the user is willing to put in the effort. There are a few exceptions. AV*i*D-6DOF does include the WGS84 Earth Gravity Model (EGM) as well as simple drag and lift models based on well-known expansion formulae for 3DOF trajectory modeling. As more cases are studied for specific research efforts more will be included in AV*i*D-6DOF to provide a library of options to choose from.

4.1 Gravity

AV*i*D-6DOF has the ability to model the complete WGS84 Earth Gravity Model (EGM) but currently has only the first 18 coefficients [11]. The user can specify any number of EGM coefficients scaling the fidelity of the gravity model down to the well-known case of spherically symmetric Newton inverse square law. There is also a symmetry setting which allows users to exclude the azimuthal dependence of the gravitational field. For the time being gravity is applied to the center of mass of each body. No attempt is made to evaluate the tidal force or gravitational gradient across an extended body. This may be added at a future time.

4.2 Aero forces

Aero forces include drag and lift. The modeling of such forces can get quite complicated and involve the modeling of air and/or other fluids as they move across the surface of the vehicle. In many cases it is sufficient to describe these effects by a set of coefficients and functions of the body kinematics. The fidelity of this aspect of the model is to a good degree determined by how much effort the user wants to place in determining these coefficients or writing external plug-in functions.

Based on the above comments AV*i*D-6DOF does not currently model explicit time dependent environmental fluctuations. The user can change look up tables from one simulation step to the next but explicit instantaneous time dependent changes are not currently supported. In a nut shell we assume that any changes in the environment occur slowly compared to the rate of vehicle movement and can be considered quasi-static and time dependence in the aero force is due to the changes in kinematics state of the moving bodies.

4.3 Thrust

Vehicle trust is modeled using a trust profile supplied in a formatted file. Thrust profiles for missiles, as an example, are expressed as acceleration versus time during the launch phase of flight [12]. For some classes of missiles this is the end of their ability to control their flight, after the launch phase they coast under the influence of gravity then in the reentry phase are acted upon by aero forces again.

To model more sophisticated flight patterns and to account for maneuvering AV*i*D-6DOF is capable of applying thrust force vectors to the motion model in response to specific input events or based on an event time table. This is used by ESSC-LLC in the modeling of satellite motion in a debris field. In their typical state a satellite in an orbital slot will rely on gravity and its inertia to keep it at a fixed altitude. Energy may be lost after several orbits due to weak atmospheric effects, changes in gravity which perturb the motion, they may maneuver to enter a graveyard orbital state or be directed to maneuver by a ground station to avoid collisions. Each of these cases is supported by AV*i*D-6DOF with the necessary data supplied by the user. Based on the parameters of the thrust model AV*i*D-6DOF handles changes body mass *etc*. internally.

4.4 Other forces and moments

AV*i*D-6DOF allows one to interface the force calculation with other external functions. A user can model forces and force fields not covered explicitly in the sections above. Furthermore one can reduce the gravity model to a constant force field, a common practice for modeling ground vehicles and low flying aero-vehicles over short distances. ESSC-LLC is engaged in realistic modeling of ship motion and other vehicles containing mounted radars and other devices for stressing the performance of said devices for detection and tracking of targets in less than ideal environments. Our initial motivation may have been Aerospace vehicle modeling but in time we anticipate Aerospace to be replaced by All in the acronym AV*i*D (All Vehicle Inclusive Design). Since we are also modeling deformation, separation and collisions all within the same simulator we may evolve the name to AV*i*D-*N*DOF (All Vehicle Inclusive Dynamic *N*-Degree of Freedom). A lofty goal, but for now we are pleased to be working in space.

5. Simulation details

This section covers a few items related to the inner working of the simulation. Since AV*i*D-6DOF is proprietary specific details on program flow, optimizations, *etc.* are not disclosed here. We introduce some high level points that we think will be of interest to the reader to illustrate some of the features of AV*i*D-6DOF that separate it from other 6DOF simulators.

AViD-6DOF implements many exact solutions for various types of motion, free rotation of symmetric and tri-inertial bodies, Kepler orbits, *etc*. Based on user inputs AV*i*D-6DOF is capable of evaluating the best method for state propagation among the available options. This is done to ensure the best possible data in the shortest amount of time. In particular users can turn the rotational motion off thus creating a 3DOF simulator.

5.1 Propagation

AV*i*D-6DOF contains a variety of propagation algorithms to accommodate fidelity and speed requirements in different situations. The equations for different degrees of freedom may have wildly disparate characteristic eigenvalues, and hence different rates or periods of motion. A standard propagator used in many simulators for ray tracing or trajectory modeling is the 4th order Runge-Kutta [13, 14]. To quote Numerical Recipes "For many scientific users, fourth-order Runge-Kutta is not just the first word on ODE integrators, but the last word as well."

For full 6DOF systems AV*i*D-6DOF uses a stiff ODE integrator. AV*i*D-6DOF also uses a variable step size and checks solutions at each step by back propagating the final conditions to recover initial conditions to within a user defined threshold. When feasible AV*i*D-6DOF will break apart

For more information contact: Dr. David R. Bergman, davidrbergman@essc-llc.com

the full state into subsystems, propagating each independently and synchronizing state variables. This allows one to use parallel processing techniques, exact solutions and other optimization techniques when appropriate.

AViD-6DOF also contains an implementation of the Lambert solution for the two point boundary value problem [15, 16]. This routine is not normally used by the propagator but is available for use in collision analysis or determining boosts required for rendezvous and orbit changes.

One can choose to forego the variable step size for a fixed step size and turn off back substitution for speed. But these choices could seriously degrade fidelity and should be done only when enough testing has revealed that solutions are trustworthy.

5.2 Coordinate systems

A detailed treatment of coordinate systems and transformations is beyond the scope of this paper. Here were merely list all the coordinate systems and transformations available to AV*i*D-6DOF. ESSC-LLC has developed a rather large and comprehensive set of coordinate systems and corresponding transformations for position, velocity and acceleration variables and their covariance matrices in support of custom tracking filter designs and testing. Since one of the main drivers for the development of AViD-6DOF is the generation of ground truth data to test these filters the coordinates are made available to the user for specifying the format of the output data.

Coordinate Systems	
Cartesian	Standard 3dim non-moving Cartesian coordinate system
Spherical	Standard non-moving spherical coordinate system
Cylindrical	Standard non-moving cylindrical coordinate system
Lat, Lon, Alt	Latitude, Longitude and Altitude defined by a spheroid
RUV	Range and two angles commonly used as array face coordinates
ECI	Earth Centered Inertial, same as Cartesian non-moving with origin at the center of the earth
ECF	Earth Centered Fixed, Cartesian coordinate system with axes fixed and rotating with the earth
ENU	East, North, Up, Cartesian coordinate system defined at a fixed point on the earth’s surface

Table 2 *Coordinates available to* AV*i*D-6DOF *for data formatting*

Transformations exist between any pair of coordinates in the above list.

6. Separation and collisions

A unique aspect of AV*i*D-6DOF is the ability to model the separation of a single body into multiple bodies and collisions between multiple bodies. There are two simplifying assumptions about the nature of collisions applied to the analysis, (1) that collisions between two bodies occur at a single point on each body and (2) that collisions occur only between pairs of bodies (three of more bodies do not simultaneous touch). More details are described in the following subsections.

6.1 Single point hard body collisions

Collision events are treated as single point hard body collisions [17-19]. Before a collision analysis is initiated bodies are checked to see if they come close enough to potentially collide within the next few steps of similar size. Each body has a spherical region around it which encompasses the body. The size of the sphere is determine by the largest distance from the CoM. Spheres are checked for overlap then steps refined for pairs of bodies that may be on a collisions course. A collision event is treated as an isolated event that occurs over a time interval very small compared to the mean step size used by the propagator. For all intents and purposes the collisions occur instantaneously and serve only reset the state to be propagated by the ODE solver.

All collision events treated by AV*i*D-6DOF conserve translational and rotational momentum and in the default state conserve energy. However the user can specify an amount of energy loss per collision and a distribution for a random draw for percent energy lost to account for different energy loss mechanisms present during a collision. Mechanical energy lost in a collision event can be converted, for example, to deformation or fracture of the bodies involved, mechanical vibrations in various elements of the bodies or heat.

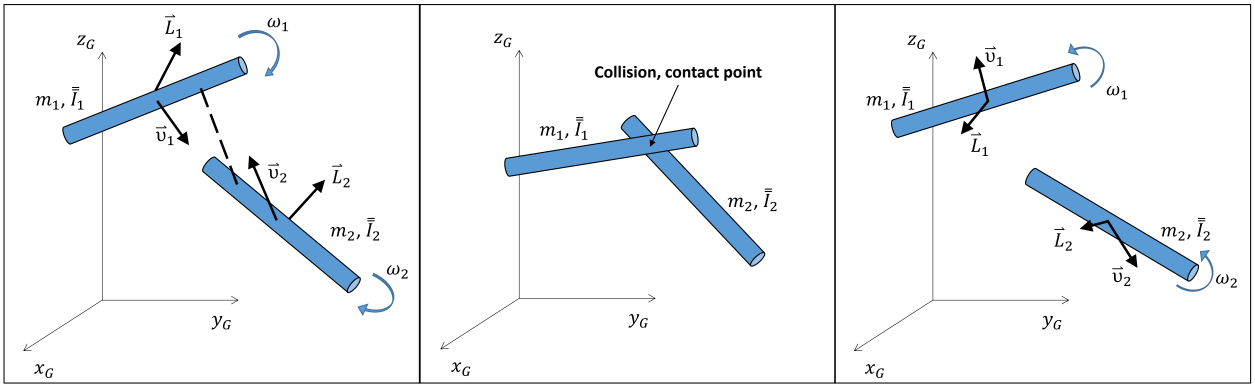

Figure 3 *Example of a collision*

Figure 4 illustrate the steps involved in a typical collision event. For illustrative purposes a notional collision between two rigid cylinders is shown. Shown with each body is its velocity, $\vec{v}_n$,

For more information contact: Dr. David R. Bergman, davidrbergman@essc-llc.com

angular rate, ω_n, angular momentum, $\vec{L}_n$, mass, m_n, and inertia tensor, $\bar{\bar{I}}_n$, $n = 1, 2$. The angular momentum vector for each body is that of the rotation of the body about its CoM, the motion of each being split into translational motion of the CoM and rotation about the CoM. The left panel of figure 4 shows the initial state of each body before collision. The dotted line is meant to indicate points on the bodies that are on a collision course. The middle panel is the collisions even and the right panel shows the bodies moments after collision.

In its current state of development AV*i*D-6DOF treats energy conserving collisions by considering the contact force to be purely in the normal direction for each body. When two bodies with smooth surfaces touch they share a common tangent plane. For collisions that do not conserve energy there is an option to allow the propagation code to allocate some portion of that non-conservative force to be in the normal direction and some to be in the tangential direction defined by the relative motion of the two surfaces. No attempt is made internally to account for the physics of slipping, sticking or traction between the two surfaces. Having the option to allocate an amount of energy loss to the normal and tangential modes of the contact force allows the user to model these effects to some degree either as a random variable or by choosing these parameters based on their own research.

AV*i*D-6DOF is able to analyze collisions for a small set of preprogrammed basic shapes, cylinder (including thin rods and flat discs), ellipsoids, and cones. If a body has a basic shape indicated as a parameter this information will be used in the analysis. Additionally if information about the surface of an object is provided for more exotic shapes AV*i*D-6DOF will perform the analysis using derived information regarding the normal field on the body surface.

6.2 Separation events

A separation event involves internal forces acting to cause a large body to break into pieces [20, 21]. The term is borrowed from the description of missile and rocket payload from a booster [22]. We use this term more generally to refer to the “breaking” of a single body into many smaller bodies. If the separation is intentional we expect a trust profile for the separation. We also model cases where the separation may be due to in internal failure, mechanical, electrical or otherwise causing a catastrophic event. To model such cases users can input a separation force profiled based on how they think the failure would occur or choose a random draw for force vector and its placement on the body. AV*i*D-6DOF will use this information to determine the force and moments acting in pairs internally to determine the final state of each body after breakup. These states are then used to initialize propagators for each body.

During a separation event translational and rotational momentum of the system are conserved, all forces and moments involved being internal. Energy may or may not be conserved. AV*i*D-6DOF can handle separation events in more than one way; (1) by assuming a gentle energy conserving separation in which child bodies inherit their states from the parent body based on constraints, (2) by applying an externally defined separation thrust profile or (3) by use of random draws to define final states, subject to appropriate constraints. The last method is ideal for investigating how certain unknowns might affect the state of a missile or vehicle.

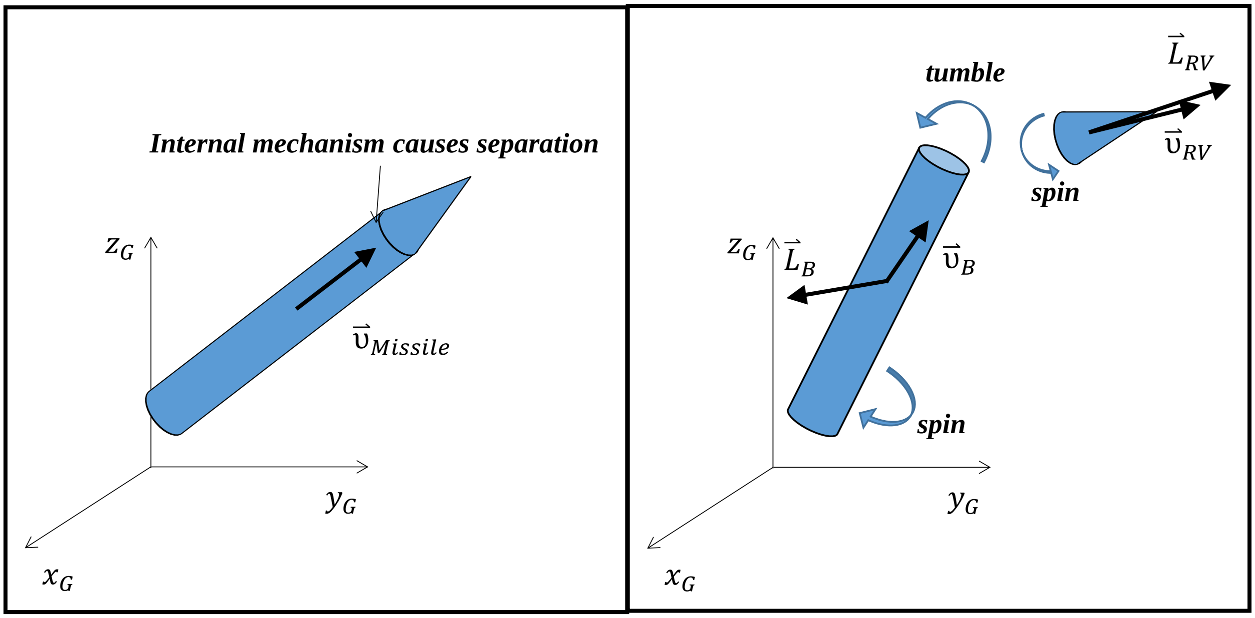

Figure 4 *Example of a separation event*

Figure 4 illustrates an example of a separation event. In this example the separation is planned and due to an internal mechanism causing a missile to separate into a booster (B) and reentry vehicle (RV). The figure is not meant to depict any particular missile, just the type of event that may occur in general. The missile is modeled as two basic shapes, a cylinder for the booster and a cone for the RV. The left panel of figure 4 shows the missile as one body with only a translational component to its motion. The right panel shows the state of the booster and RV moments after separation where both have rotational components to their motion.

Final Remarks

The majority of ESSC-LLC research to date has been in the study of missile and satellite motion during various phases of flight, the break-up of missiles, and the interaction of these objects with debris fields. Much of this research has been in support of developing ground truth data to feed radar cross section simulations and in the development of custom tracking filters. AV*i*D-6DOF developed in time to encompass more physics and provide greater complexity and fidelity as the need presented itself and is an evolving simulation tool.

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Contact Information

Dr. David R. Bergman, Chief Scientist
Exact Solution Scientific Consulting LLC

Mailing Address:
Picatinny Arsenal Technology Innovation Center
3159 Schrader Road
Room 122
Dover NJ 07801

Phone/email
(551) 655-4720
davidrbergman@essc-llc.com